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| **Posing Powerful Questions** |

**Lesson Title: The Border Problem Grade 9 Applied**

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| **Goals(s) for a Specific Lesson**   * Students will solve problems by creating algebraic models and compare different solutions. |
| **Curriculum Expectations**   * Students will connect various representations of a linear relation. * Students will manipulate numerical and polynomial expressions and solve linear equations. |
| **Big Idea(s) Addressed by the Expectations**  Algebraic reasoning is a way to understand mathematical relationships that apply to a large group of situations. |
| **Minds On…**  Students will practice their simplification skills to show that the models they built for the toothpick problem (TIPS 7.4.4) are all equivalent. |
| **Action!**  A company manufactures and installs swimming pools for resorts and recreation centres. To complete the installation, the company will create a border of 1m by 1m tiles around the pool. The company president wants you to determine a formula for the number of tiles required for any size square pool. |
| **Scaffolding Questions**   * Would drawing a diagram of the problem help you? * How many tiles would it take to border a pool with a width of 1m, 3m, 5m? * Have you considered summarizing your findings in a table and looking for patterns? * How does the width of the border differ from the width of the pool? |
| **Consolidate/Debrief:** Students share their solutions through a math congress, explaining the reasoning behind the formula they chose.  **Common Questions:**   * Why are there a variety of different formulas that correctly answer the question? * How could algebra be used to convince you that all of these formulas are the same? |

**Some Possible Solutions**

Solution #1: Consider some smaller pools …

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Create a table to compare the number of tiles needed for each different size of pool …

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| Pool Size  (width ***n***) | Number of Tiles (**T**) |
| 1 | 8 |
| 2 | 12 |
| 3 | 16 |
| 4 |  |
| 5 |  |

Look for patterns … what do you see?

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| Pool Size  (width ***n***) | Number of Tiles (**T**) |  |
| 1 | 8 | 4(2) 🡪 4(1 + 1) | |
| 2 | 12 | 4(3) 🡪 4(2 + 1) | |
| 3 | 16 | 4(4) 🡪 (3 + 1) | |
| 4 | 20 | 4(5) 🡪 (4 + 1) | |
| 5 | 24 | 4(6) 🡪 (5 + 1) | |

observation: The number of tiles is always 4 times the number that is 1 more than the width of the pool.

formula: T = 4(n + 1)

Solution #2: When I look at the pool border, I see that the number of tiles for the border is the same as the number of tiles around the perimeter plus the 4 extra corner tiles.

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So, the number of tiles for the border of this pool is: T = 4(10) + 4 = 44

The formula for the number of tiles in the border of a pool with width ***n*** is: **T =** 4**n** + 4

Solution #3: When I look at the pool border, I see that the top and bottom rows of the border each contain two tiles more than the width of the pool. The side borders are each the same size as the width of the pool.

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The number of tiles for this pool is: T = 2(10 + 2) + 2(10) = 44

The formula for the number of tiles in the border of a pool with width ***n*** is: **T** = 2(**n** + 2) + 2**n**

Solution #4: When I look at the pool border I see the border composed of four rows of tiles, one on each side of the pool. Each row of tiles is 1 tile longer than the width of the pool.

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The number of tiles in the border of this pool is: **T** = 4(10 + 1) = 44

The formula for the number of tiles in the border of a pool with width ***n*** is: **T** = 4(**n** + 1)

Solution #5: When I look at the pool border I see two squares … the larger square represents the border and the smaller square is just the pool itself. The number of tiles in the border is just the number of tiles in the larger square minus the number of tiles in the smaller square.

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The larger square is two rows wider than the smaller square. So, **T** = (10 + 2)2 – 102 = 44

The formula for the number of tiles in the border of a pool with width ***n*** is: **T** = (**n** + 2)2 – **n**2